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The interplay between reputation and heterogeneous investment enhances cooperation in spatial public goods game

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ABSTRACT

In previous studies about heterogeneous investment, many researchers only focus on the monodirectional effect of reputation, but ignore the reversed influence about the heterogeneity of reputation fluctuation driven by investment. Given this, considering the interplay between reputation and heterogeneous investment, we explore the evolution of cooperation in public goods game. In detail, each player's reputation situations in different groups determine its differentiated investment amounts, and in turn, the heterogeneity of reputation fluctuation is dependent on how much it invests into each group. Furthermore, we introduce two parameters, α and β, to characterize the effect degree of investment on heterogeneous reputation fluctuation and the one of reputation on heterogeneous investment, respectively. The simulation results suggest that cooperation can be boosted to some extent, especially for the larger α and β, as they magnify the interplay of reputation and heterogeneous investment.

1. Introduction

Collective cooperation is helpful to obtain the optimal payoffs for the population, while rational individuals maximize their own benefits by defection. The huge conflict between the optimal payoffs of population and individuals results in social dilemma [1–4]. However, cooperative behavior is omnipresent. Therefore, the explanation for the emergence of cooperation in social dilemma remains an open issue and has been paying much attention of scholars in many fields. Evolutionary game theory [5–7] is utilized into many game theoretical approaches, such as prisoner's dilemma game (PDG) [8–12], snowdrift game (SDG) [13–15], and public goods game (PGG) [16–18], to investigate how to boost the evolutionary cooperation. Among them, PGG is a classical model to focus on the cooperative behavior of group interactions.

In PGG, players decide whether or not to contribute to the common pool, and the total investment is multiplied and allocated by all the players evenly. As a consequence, cooperators who pay cost are free-ride by defectors without any contribution, under which "the tragedy of commons" comes into being. Up to date, there are numerous studies about PGG on structural network, including square lattice [19–21], scale-free networks [22–24], interdependent network [25,26], and so on. Moreover, many mechanisms, such as reputation [27–30], exclusion [31,32], persistence [33–35], emotion [36–38] and heterogeneous investment [39–41] have been proposed for the purpose of investigating the evolution of cooperation.

In most studies, cooperators contribute the consistent investment into all the groups they participate in, while diversity of contribution or heterogeneous investment in accordance with the reality cases, where individuals invest distinct cost into different items, is also reasonable. Therefore, the heterogeneous investment mechanism is verified by many researches and they find the cooperation can be facilitated. The effects of heterogeneous contribution driven by the groups' cooperation situation on the evolution of cooperation were explored in [40,41]. And the influences of structural network, such as the scale-free networks were also demonstrated in the follow-up studies [42,43]. Furthermore, in subsequent studies, many scholars take the heterogeneity of investment induced by other mechanisms into consideration. Lv et al. studied the impact of heterogeneous investments on cooperative behavior in public goods game with exclusion [44]. The interactions between the heterogeneous investment and synergy factor were proposed by Weng et al. [45]. Zhang et al. introduced the heterogeneous investments induced by historical payoffs into spatial PGG [46]. Reputation-based heterogeneous investment were investigated by Yang et al. [47] and Ma et al. [48], respectively. In the former studies, they assumed that the heterogeneity of contributions depends on the reputation of group leader and players invest more into the group with higher leader reputation. And for the latter research, a threshold was taken into consideration. On the one hand, once all of the remaining members' reputation values of the identical group reach

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the threshold, the cooperator will contribute one unit to the group. On the other hand, cooperators adjust dynamically the investment according to the number of players reaching the reputation threshold.

In the aforementioned studies about reputation-based heterogeneous investment, the interaction between reputation and investment is monodirectional, in which reputation unilaterally influences the heterogeneity of investment, but not be influenced in turn. In addition, each player only has one reputation value, and the variation of it is totally dependent on the strategy one chooses. In detail, the fluctuation scale of reputation is fixed, the value increases by 1 for cooperation, and decreases by 1 for defection. However, in fact, besides the influence of reputation on the heterogeneous investment, in turn, the effect of investment on the reputation should also be thought about. Such instructions can explain it. On account of the players' heterogeneous behavior, which is also called unequal investment, other members' evaluation in distinct groups about the player generates the differentiation. As a consequence, some attributes of players, such as reputation, should also be distinguished in different groups. In parallel, how much the cooperative player invests into the identical group determines the fluctuation scale of the player's reputation, a more investment gives rise to a higher increment of its reputation. In other words, the reputation of the same player is heterogeneous in distinct groups, and the fluctuation scale is dynamic. Thus the interplay between heterogeneous reputation fluctuation and unequal investment is taken into account. In general, one inclines to invest much more into the group in which other members have higher reputation for the sake of free riding. Nevertheless, it is able to obtain higher reputation for more investment. Thereout, although the investment way inclines directly to cooperation, the introduction of the heterogeneous reputation fluctuation restricts the obvious consequence as the apparently increased reputation leads to the less investment at the next time step. And we call this special mechanism as reciprocal restriction of reputation and investment. To explore the evolution of cooperation in such a situation, we apply the mutual restriction of reputation and heterogeneous investment into spatial public goods game. To be specific, each player has corresponding reputation value of each group they participate in, and it decides how much investment share to contribute according to the proportion of its own reputation in the group's reputation. Meanwhile, in turn, the variation of reputation is influenced by the investment share contributed into each group. The simulation results suggest that under such a mutual restriction of reputation and heterogeneous investment mechanism, cooperation can be surprisingly facilitated, although the effect is marginal.

The following instructions explain the arrangement of the remaining contents in this work. The expatiation of the whole model can be found in Section 2, and the simulation results are presented and discussed in Section 3. At last, we make some conclusions in Section 4.

2. Model

A square lattice with periodic boundary conditions composed of $L*L$ nodes is considered in this model. Each player is located on the node and interacts with its four nearest neighbors. Thus, each player engages in 5 different groups which centered by itself and its direct neighbors to play a PGG of 5 players. Each player has two choices, to be a cooperator or a defector. And in every group the player participates, it has a reputation to reflect the experience of its contribution in this group. Initially, each player is set to be a cooperator ($s_i = 1$) or a defector ($s_i = 0$) equally. And each player is distributed a random reputation value from 1 to 100, and particularly, the five reputation value of one is consistent.

Herein, we denote the player i's investment and reputation in the group organized by player j at t time step as $I_{ij}(t)$, and $R_{ij}(t)$, respectively. At each time step, a defective player contributes nothing, while for a cooperator, the investment paid into five groups is heterogeneous, and it depends on its reputation in the corresponding group. And thus, $I_{ij}(t)$ can be defined as follows,

$$I_{ij}(t) = (1 - \frac{R_{ij}(t)}{\sum_{x\in\Omega_j} R_{xj}(t)})^{\beta} s_i(t), \tag{1}$$

where Ω_j represents the set of player j and its four neighbors, and β is an adjustable parameter which ranges from 0 to 1. Particularly, when $\beta = 0$, the model turns into the traditional PGG in which cooperators invest 1 to all the groups.

In turn, reputation updating is dependent on the players' investment. A defector's reputation fluctuation is consistent in five groups, and all the reputation value decreases by 1. Moreover, for a cooperative player, the heterogeneity of investment influences the fluctuation scale of its reputation. More investment of one in the group leads to the higher reputation variation in an identical group. Therefore, reputation updates according to the following formula,

$$R_{ij}(t+1) = \begin{cases} R_{ij}(t) + (\frac{I_{ij}(t)}{\sum_{x\in\Omega_j} I_{xj}(t)})^{\alpha}, & s_i(t) = 1 \\ R_{ij}(t) - 1, & s_i(t) = 0 \end{cases}. \tag{2}$$

Herein, α is an adjustable parameter with value ranging from 0 to 1. In particular, when $\alpha = 0$, the variation of a cooperator's reputation is homogeneous, which equals to 1. And only if the following two conditions are met, can the interplay between heterogeneity of reputation and investment come into being, that is $\alpha > 0$ and $\beta > 0$.

The total contribution is multiplied by a synergy factor r, and then allocated by all the group members evenly no matter whether to invest or not. Hence, the payoff of player i obtained from the group centered by player j is,

$$\pi_{ij}(t) = \frac{r \sum_{x\in\Omega_j} s_x(t) I_{xj}(t)}{5} - s_i(t) I_{ij}(t), \tag{3}$$

and the total payoff that player i gains in all five groups is given by the following formula,

$$\Pi_i(t) = \sum_{j\in\Omega_i} \pi_{ij}(t), \tag{4}$$

where Ω_i denotes the set of player i and its four nearest neighbors.

In quick succession, players update their strategy synchronously by imitating someone else. One of the nearest neighbors is selected randomly, and then the focal player imitates the chosen player's current strategy to be its strategy at next time step with the following probability,

$$W(S_j \to S_i) = \frac{1}{1 + \exp[(\Pi_i - \Pi_j)/K]}, \tag{5}$$

where K characterizes the intensity of noise and in this paper, and we set $K = 0.1$. It is worthy to notice that the introduction of reputation only influences the unequal investment, but not the strategy updating rule.

The whole evolutionary process is obtained by simulating a sufficiently long time (10^4 steps) on a square lattice of $100*100$ nodes, and the stable cooperation level is averaged by last 1000 time steps. For precision, we run more than 20 independent realizations and the results of the following graphs are obtained by the average.

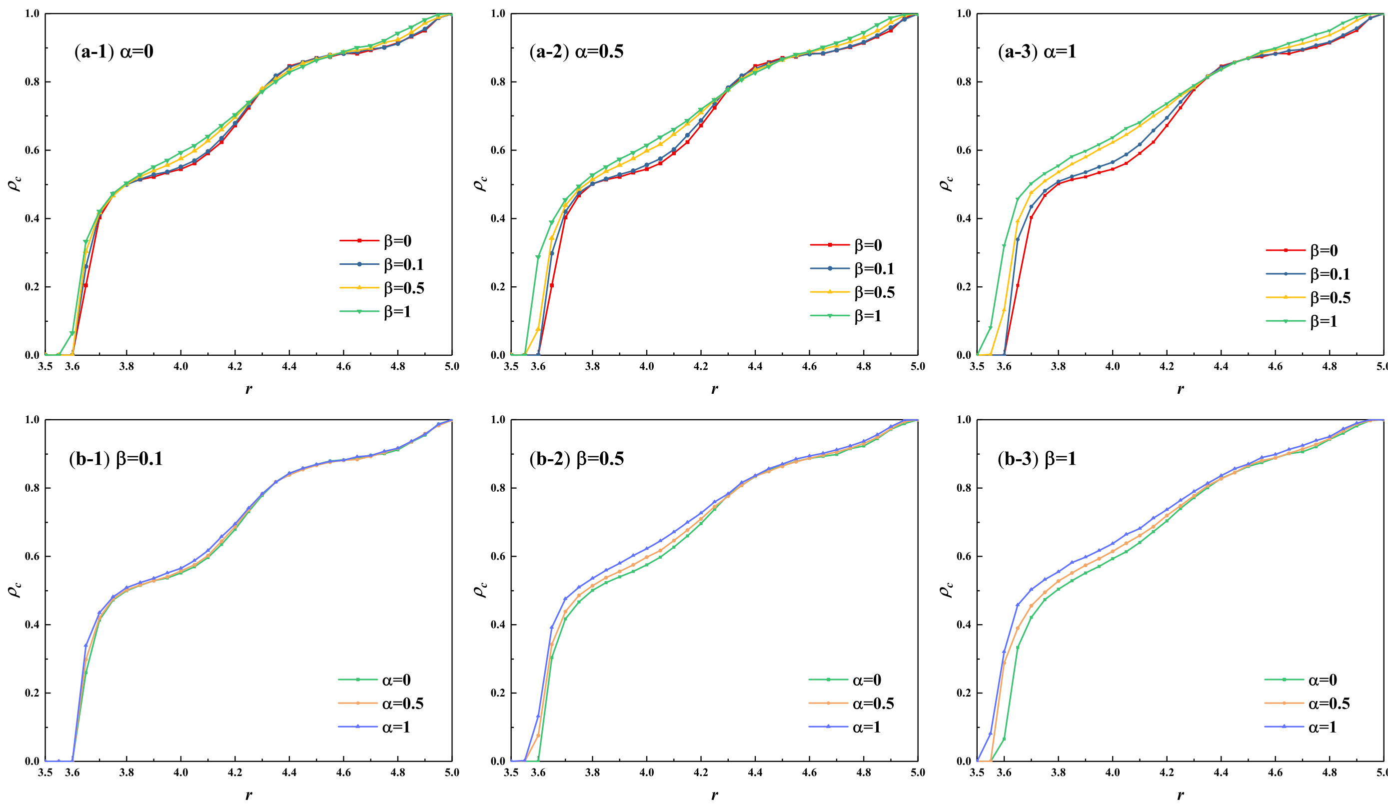

Fig. 1. The effects of the parameters β and α on the proportion of cooperators with the variation of synergy factor r. The detail parameter settings are as follows. Different values of β, including $\beta = 0$, 0.1, 0.5, and 1 in (a-1)$\alpha = 0$, (a-2)$\alpha = 0.5$, (a-3)$\alpha = 1$, and different values of α, including $\alpha = 0$, 0.5, and 1 in (b-1)$\beta = 0.1$, (b-2)$\beta = 0.5$, (b-3)$\beta = 1$.

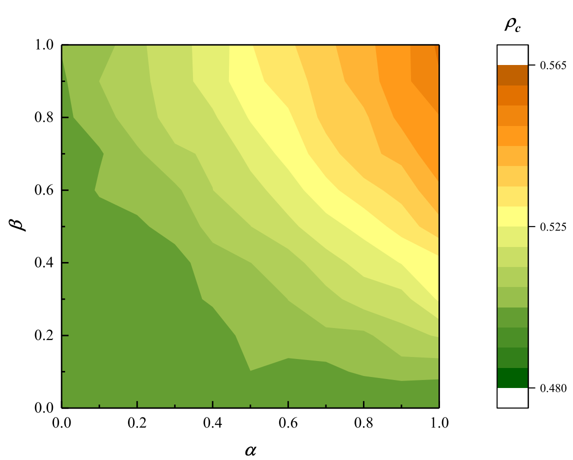

Fig. 2. The effects of the adjustable parameter α and β on the fraction of cooperators when the synergy factor is set to be 3.8.

3. Results and discussion

For the purpose of investigating the effects of the model, which considers the interplay between reputation and unequal investment, we plot curves in Fig. 1 and the heat map in Fig. 2 to reveal the variation of cooperation level. Apparently, on the whole, we can observe that although the consequence of the model is marginal, it is positive. The reason for the marginal consequence is that the introduction of the reciprocal restriction between reputation and heterogeneous investment. On the one hand, more investment into the group with other members' higher reputation is notably helpful to foster cooperation. However, on the other hand, in our model, we also consider the conversed effect of heterogeneous investment on the reputation fluctuation. The more investment would generate higher reputation in the identical group. And the apparently increased reputation restricts the higher investment share at next time step. Therefore, what is attracting and surprising is that under such a mutual restriction mechanism, the cooperation can be promoted, although the effect is not in a big scale. Next, we make some detailed instruction and analysis. In Fig. 1, as usual, the cooperation level goes up with the weakening dilemma strength. In our model, $\beta = 0$ is corresponding to the traditional PGG, and thus all the results in Fig. 1 (a) suggest that our model has some effects in developing cooperation, and the effect with a higher value of β is more apparent no matter what value α takes. In addition, $\alpha = 0$ corresponds to the general variation of reputation in previous models, the homogeneity of reputation variation, in which cooperation leads to the reputation to increase by 1, and defection results in the decreasing reputation value by 1. Therefore, the sub-graphs of Fig. 1(b) exhibit the advantages of heterogeneous fluctuation scale of reputation. At the same time, under the cases of $\alpha = 0.5$ and 1, the higher cooperation level than the case of $\alpha = 0$ also verifies that the potential on boosting cooperation of the interplay between reputation and unequal investment is larger than the one generated by the monodirectional effect of reputation on heterogeneous contribution. Concretely speaking, a bigger α magnifies the differences of the effect on facilitating cooperation among different β, and in turn, similarly, the cooperation level can be boosted more effectively with a larger α regardless of the β values, and the effect differences among distinct α are amplified by a larger β. In particular, we find that the consequence of the two parameters α and β is relatively more apparent when the synergy factor r takes the medium value. Thus, in Fig. 2, the introduction of the heat map depicting the comprehensive effects of the adjustable parameter α and β on the fraction of cooperators does much to further demonstration about the effects of the

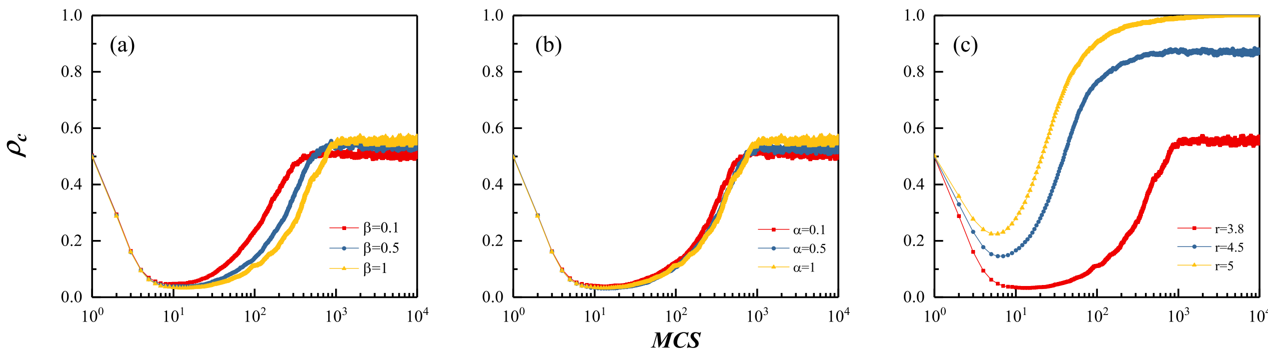

Fig. 3. Time series of the whole evolutionary process for different r, α and β settings.

two parameters when r is fixed at 3.8. Lengthways, the gradually elevated cooperation situation with the enlarging β proves that the strength of the reputation-based unequal investment, and in turn, the lateral lifted results, except the case when $\beta = 0$, with the increasing α suggests that the feasibility of the heterogeneous reputation fluctuation which is based on the unequal investment. Therefore, combining Fig. 2 with Fig. 1, we can conclude unhesitatingly that the synchronous larger values of α and β play a more significant role in improving cooperation.

Fig. 3 is depicted to give a visual representation about the evolutionary time series of cooperation for distinct cases of the parameters. The particular settings are as presented below, $r = 3.8$, $\alpha = 1$ in (a), $r = 3.8$, $\beta = 1$ in (b) and $\alpha = 1$, $\beta = 1$ in (c). First, the entire curves have the completely accordant evolution tendency. Under the enticement of defective higher benefits, the proportion of cooperative players reduces in the enduring period (END) [49]. Subsequently, the potential of the model appears gradually, hence cooperators increase during the expanding period (EXP) [50] until the final cooperation level holds steady. Furthermore, three subgraphs also exhibit the effects of the three parameters, separately. In Fig. 3(a), although a larger β decelerates the speed of reaching the steady state, it amplifies the result of steady state. Similar as β, the effect of parameter α is able to be concluded in Fig. 3(b). Furthermore, obviously, the effect of synergy factor r in Fig. 3(c) is definitely coincident with common sense. It not only accelerates the speed of achieving the final result, but also magnifies the steady cooperation level.

In order to further demonstrate the effectiveness of the mechanism, we plot the histograms of cooperators proportion within distinct intervals which is dependent on the maximal difference among its five investments in Fig. 4(a) and the maximal difference among its five reputation values in Fig. 4(b) when $MCS = 10000$. For the reason that the unequal investment is exclusively personal to the cooperative players, the ordinate value is obtained from the fraction of the corresponding players divided by the fraction of entire cooperators. In Fig. 4(a), the investment is divided into 10 intervals and the histograms are obtained when α is fixed at 1. The red histograms and blue ones represent the results when $\beta = 0.1$ and $\beta = 1$, respectively. The two sub-graphs of Fig. 4(a), $r = 3.8$ in (a-1), $r = 4.2$ in (a-2) all testify more clearly the influence of the parameter β which magnifies the effects of the heterogeneous investment, as we can obviously find that blue histograms are more widely distributed than the red ones regardless of the values of the synergy factor and even throughout the whole intervals. Similarly, the histograms in Fig. 4(b) are obtained when β is fixed as 1, and they represent the results when r=3.8 in (b-1) and r=4.2 in (b-2), respectively. And the reputation is also divided into 10 intervals, from 0 to 100. The yellow and green histograms plot the distribution of $\alpha = 0.1$ and $\alpha = 1$, respectively. The more widely distributed green histograms verify the consequence of parameter α intuitively which denotes the influence of the heterogeneous reputation variation driven by the unequal investment and a larger α amplifies the influence.

4. Conclusions

In most previous studies about heterogeneity in public goods game, there exist many considering heterogeneous investment driven by reputation. In all of them, cooperative players obtain the increased reputation by 1 regardless of how much they invest into distinct groups. Nevertheless, the distinct reputation in different group for the identical player is totally reasonable. Meanwhile, it is justifiable to introduce the heterogeneity of reputation fluctuation because reputation not only determines one's investment, but also is affected by investment in turn in reality. As usual, for the sake of free-riding, one is willing to invest more into the group with higher other members' reputation, it is apparently useful for the promotion of cooperation. But added with the conversed influence of investment on the reputation, the obviously increased reputation due to the more investment would restrict the next investment in the same group. Therefore, the introduction of the reciprocal restriction between reputation and heterogeneous investment leads to the expected consequences disappeared and in order to explore the effect under such a particular mechanism, we apply the interplay between reputation and heterogeneous investment into the evolutionary public goods game, and surprisingly find that the cooperation can be maintained and developed, although the effect is not in a big scale. In future studies, other network structures can be introduced and analyzed. Our work compensates for the gaps of the unidirectional effect of reputation on unequal investment in past researches, and certifies the necessity and feasibility of considering the complex perception of human beings, such as the heterogeneous reputation variation originated from the other members' differentiated assessments of identical person, and provides more possibility to further comprehend the evolution of cooperation in social dilemma.

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CRediT authorship contribution statement

Jialu He: Writing – original draft, Methodology, Formal analysis, Data curation. **Jianwei Wang:** Supervision, Project administration, Funding acquisition. **Fengyuan Yu:** Software, Investigation, Conceptualization. **Wei Chen:** Writing – review & editing, Validation, Resources. **Yuhao Ji:** Writing – original draft, Visualization, Formal analysis.

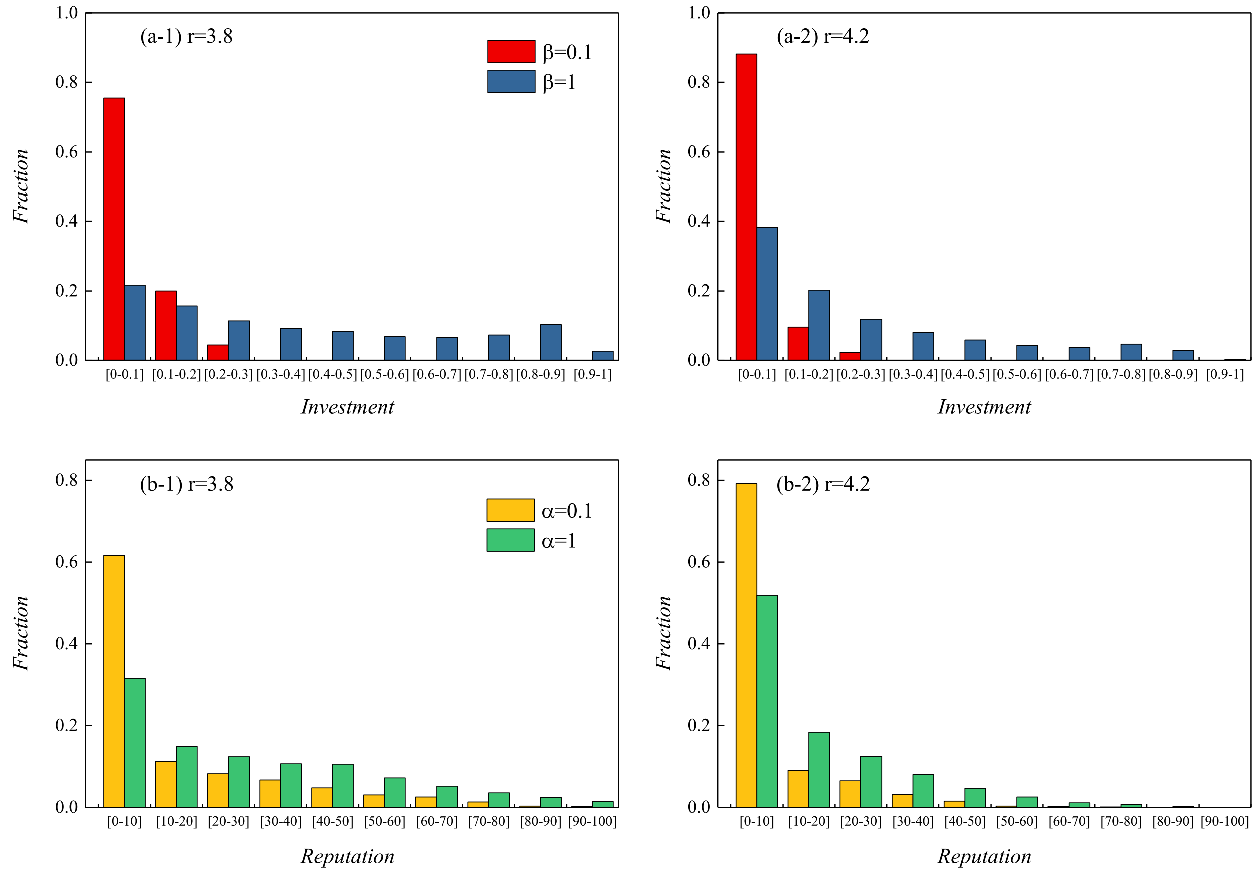

Fig. 4. Distribution situation of cooperative players within distinct intervals of the maximal difference of investment (a) or reputation (b) when $MCS = 10000$.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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